Nonequilibrium phase transition in the kinetic Ising model driven by propagating magnetic field wave

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The two dimensional ferromagnetic Ising model in the presence of a propagating magnetic field wave (with well defined frequency and wavelength) is studied by Mone Carlo simulation. *This study differs from all of the earlier studies done so far, where the oscillating magnetic field was considered to be uniform in space.* The time average magnetisation over a full cycle (the time period) of the propagating magnetic field acts as the dynamic order parameter. The dynamical phase transition is observed. The temperature variation of the dynamic order parameter, the mean square deviation of the dynamic order parameter, the dynamic specific heat and the derivative of the dynamic order parameter are studied. The mean square deviation of the dynamic order parameter, dynamic specific heat show sharp maxima near the transition point. The derivative of dynamic order parameter shows sharp minimum near the transition point. The transition temperature is found to depend also on the speed of propagation of the magnetic field wave.

Keywords: Ising model, Monte Carlo simulation, Dynamic transition, Propagating wave
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1 Introduction:

The nonequilibrium response of Ising ferromagnet in the presence of time varying magnetic field is widely studied [1]. Among all these dynamical responses (e.g., hysteretic response, dynamic phase transition, stochastic resonance etc.), the nonequilibrium dynamical phase transition is an important phenomenon[1] and became an interesting field of research recently. These dynamic phase transition has several similarities with that observed in the case of equilibrium thermodynamic phase transition. The effort in studying the invariance of time scale (i.e., critical slowing down) [2], the divergence of specific heat [2], divergence of critical fluctuations in energy [3], divergence of length scale near the transition point [4], the order of the transition [5] established the dynamic transition as an interesting nonequilibrium phase transition. This dynamic transition is very closely related to the hysteretic loss[6] and the stochastic resonance[7]. Experimentally the existence of dynamic transition was found [8] in Co film on Cu surface (at room temperature) by surface magneto optic Kerr effect. Recently, the evidence of dynamic phase transition was found expreimentally [9], in $[\mathrm{Co}(4\text{Å})\mathrm{Pt}(7\text{Å})]_3$ multilayer system with strong perpendicular anisotropy by applying a time varying (sawtooth type) out-of-plane magnetic field in the presence of small additional constant magnetic field. In this study, the dynamic phase boundary was drawn and found similar to that obtained from the simulation in kinetic Ising model with analogous condition.

The dynamic phase transition is also observed[10, 11, 12, 13] in other ferromagnetic models. It is studied[14] in the Ginzburg-Landau model of anisotropic XY ferromagnet and different types of chaotic behaviour is observed. Recently, the multiple dynamic transitions is observed[15, 16] in anisotropic Heisenberg model. These studies are reviewed[17] recently.

However, all these studies, done so far for the dynamic phase transition are made with time varying magnetic field which was uniform over the space. No attempt has been made to study the dynamic phase transition with magnetic field depending on both space and time. In this article, the dynamic phase transition is studied, by Monte Carlo simulation[18], in Ising ferromagnet in the presence of a propagating magnetic field wave.

This article is organised as follows: the next section is devoted to describe the model and the Monte Carlo simulation method. The simulation results are reported in section -3 and the article ends with a summary in section -4.

2 Model and Simulation:

The Hamiltonian, of an Ising model (with ferromagnetic nearest neighbor interaction) defined in two dimensions (square lattice) in the presence of a propagating magnetic field wave, can be represented as

$$H = -J \sum_{<ij>} s_i s_j - \sum_i h(\vec{r}, t) s_i. \qquad (1)$$

Here, $s_i(=\pm 1)$ is the Ising spin variable, $J(>0)$ is the ferromagnetic interaction strength and $h(\vec{r},t)$ is the value of the propagating magnetic field wave at any time t and at position $\vec{r}$. Here, the propagating magnetic field ($h(\vec{r},t)$) wave is represented as

$$h(\vec{r},t) = h_0 \cos(\omega t - Ky) \tag{2}$$

where h_0 is the amplitude and $\omega(=2\pi f)$ is the angular frequency of the oscillating field and $K(=2\pi/\lambda)$ is the wave vector. Here, f is the frequency and λ is the wavelength (measured in the unit of lattice spacing) of the propagating magnetic field wave. The wavelength, considered here, is smaller than and commensurate with the lattice size (L). Here, the direction of propagation of the magnetic field wave ($h(y,t)$) is taken along the y direction only. *It may be noted here, that all earlier studies of the dynamical phase transitions are done with oscillating (in time) but uniform (over the space) magnetic field.* The boundary condition is taken periodic in all directions. This completes the description of the model.

In the simulation, the system is cooled gradually from a high temperature. Randomly selected 50% up ($s_i = +1$) spins, is taken as the initial configuration. Physically, this corresponds to the high temperature configuration of spins. In the cooling process, the last spin configuration corresponding to a particular temperature was used as the initial configuration of next lower temperature. At any finite temperature T, the dynamics of this system has been studied here by Monte Carlo simulation using Metropolis single spin-flip rate [18]. The transition rate is specified as

$$W(s_i \to -s_i) = \mathrm{Min}\,[1, \exp(-\Delta H/k_B T)] \tag{3}$$

where ΔH is the change in energy due to spin flip ($s_i \to -s_i$) and k_B is the Boltzmann constant. Any lattice site is chosen randomly and the spin variable (s_i) is updated according to the Metropolis spin flip probability. L^2 such updates constitute the unit (Monte Carlo step per spin or MCSS) of time here. The instantaneous bulk magnetisation (per site), $m(t) = (1/L^2)\sum_i s_i$ has been calculated. The time averaged (over the complete cycle of the propagating magnetic field wave) magnetisation,

$$Q = \frac{1}{\tau} \oint m(t) dt, \tag{4}$$

defines the dynamic order parameter[1]. The frequency is $f = 0.01$ (kept fixed throughout the study). So, one complete cycle of the propagating field takes 100 MCSS (time period $\tau = \frac{1}{f} = 100$ MCSS). A time series of magnetisation $m(t)$ has been generated up to 2×10^5 MCSS. This time series contains 2×10^3 (since $\tau = 100$ MCSS) number of cycles of the oscillating field. Here, first 10^3 numbers of such transient values are discarded to get the stable values of the dynamical quantities. The dynamic order parameter Q has been calculated over 10^3 values. It is checked (for a few data) that these number of samples (N_s) is sufficient to get the stable values of the dynamical quantities. So, the statistics (distribution of Q) is based on $N_s = 10^3$ different values of Q. To have the confidence (with these number of samples), the mean square deviation (i.e., $<(\delta Q)^2> = <Q^2> - <Q>^2$)

of Q is also calculated and studied as a function of temperature. It may be noted here, that values of the dynamic order parameter (at lower temperatures) become both positive and negative with equal probability. Here, only the positive values of Q are shown. The statistical error (Δ) in calculating Q may be defined as the square root of $< (\delta Q)^2 >$. The maximum error (Δ_{max}) occurs near the transition point and this reasonably indicates the critical fluctuations.

The time average dynamic energy is defined as

$$E = \frac{1}{\tau} \oint H dt. \tag{5}$$

The dynamic specific heat ($C = \frac{dE}{dT}$) is also calculated. The temperature variations of all these (above mentioned) quantities are studied.

Here, the temperature T is measures in the unit of J/k_B, the field amplitude h_0 and energy E are measured in the unit of J.

3 Results:

To investigate the nature of the spatio-temporal variations of field $h(y,t)$ and the local 'strip magnetisation', $m(y,t)$ ($= \int \frac{s(x,y)}{L} dx$, where $s(x,y) = \pm 1$ is the spin variable at position (x,y)), are studied as a function of coordinate y (along the direction of propagation of field wave) for different times (t). Fig-1, shows such plots. From the figure, the propagating nature of the field wave and the 'strip magnetisation' is clear. Here, it may be noted that, for a particular instant of time, the magnetic field and the 'strip magnetisation' differ by a phase. It is observed that, this phase difference depends on temperature of the system, wavelength and the frequency of the propagating magnetic field wave. The systematic study of this dependence requires lot of computational effort and time.

The temperature variation of the dynamic order parameter is studied. This is shown in Fig-2. For the fixed values of the amplitude, frequency and the wavelength of the propagating magnetic field wave, it is observed that below a certain temperature the dynamic ordering develops ($Q \neq 0$) and vanishes ($Q = 0$) above it. Keeping the values of frequency (f) and the wavelength (λ), of the propagating magnetic field wave, fixed, if the amplitude (h_0) of the field increases the dynamic phase transition occurs at lower temperature. For comparison, a similar studies are done for nonpropagating (sinusoidally oscillating in time but uniform over the space) magnetic field with same frequency and amplitude. This clearly indicates that the dynamic transition occurs at different higher temperatures than observed in the case of a propagating field.

These dynamic transition temperatures can be estimated by studying the temperature variations of the mean square fluctuations ($< \delta Q^2 >$) of the dynamic order parameter Q. These results are shown in Fig-3. Here, the $< \delta Q^2 >$ shows very sharp maximum, indicating the dynamic transition temperature. From this one can estimate the maximum error (Δ_{max}) involved in statistical calculation for the dynamic order parameter Q. For

propagating magnetic field wave of $f = 0.01$ and $\lambda = 25$, the dynamic phase transitions (indicated by the maxima of $< \delta Q^2 >$) occur at $T = 1.50$ and $T = 1.88$ for the field amplitudes $h_0 = 0.5$ and $h_0 = 0.3$ respectively. Here also, for comparison, the similar studies are done in the case of nonpropagating magnetic field. Here, for $f = 0.01$ the dynamic phase transitions occur at $T = 1.68$ and $T = 1.94$ for $h_0 = 0.5$ and $h_0 = 0.3$ respectively.

The derivative ($\frac{dQ}{dT}$) of the dynamic order parameter Q is calculated by central difference formula[19]

$$\frac{dQ}{dT} = \frac{Q(T + \Delta T) - Q(T - \Delta T)}{2\Delta T}. \tag{6}$$

In the simulation, the system was being cooled from a high temperature (random spin configuration) to a certain temperature slowly in the step $\Delta T = 0.02$. It may be noted here that the error in calculating the derivative numerically by this central difference formula is $O((\Delta T)^2)$[19]. So, the error involved is of the order of 0.0004. The temperature variation of the derivative of the dynamic order parameter is studied and the results are shown in Fig-4. Here, the derivative shows very sharp minimum, indicating the dynamic phase transition temperature. For propagating magnetic field wave of $f = 0.01$ and $\lambda = 25$, the dynamic phase transitions (indicated by very sharp minima of $\frac{dQ}{dT}$) are observed to occur at $T = 1.50$ and $T = 1.88$ for the field amplitudes $h_0 = 0.5$ and $h_0 = 0.3$ respectively. For a comparison, the similar studies are done in the case of nonpropagating magnetic field. Here, for $f = 0.01$ the dynamic phase transitions occur at $T = 1.68$ and $T = 1.94$ for $h_0 = 0.5$ and $h_0 = 0.3$ respectively.

The dynamic specific heat (C) is calculated from the derivative ($\frac{dE}{dT}$) of dynamic energy (E). Here also, the derivative is calculated by using central difference formula (described above). The results are shown in Fig-5. The specific heat becomes maximum near the dynamic transition point indicating the dynamic transition independently. Here, the term *independently* means the following: Here, the dynamic phase transition is studied and the transition temperature is estimated from two types of quantities. One is dynamic order parameter Q and its derivatives ($\frac{dQ}{dT}$), moments ($< \delta Q^2 >$) etc. These depend directly on Q. Another quantity is dynamic specific heat ($C = \frac{dE}{dT}$), which is not directly related to Q. For propagating magnetic field wave of $f = 0.01$ and $\lambda = 25$, the dynamic phase transitions (indicated by the maxima of $C = \frac{dE}{dT}$) occur at $T = 1.50$ and $T = 1.88$ for the field amplitudes $h_0 = 0.5$ and $h_0 = 0.3$ respectively. Here also, for comparison, the similar studies are done in the case of nonpropagating magnetic field. For $f = 0.01$ the dynamic phase transitions occur at $T = 1.68$ and $T = 1.94$ for $h_0 = 0.5$ and $h_0 = 0.3$ respectively.

The dependence of the dynamic phase transition, on the speed of propagation of the propagating magnetic field, is studied briefly. Here, for $f = 0.01$, $h_0 = 0.5$ the temperature variations of the dynamic order parameters for $\lambda = 25$ and $\lambda = 50$ are studied. The results are shown in Fig-6. It is observed that the dynamic transition occurs at higher temperature for higher speed ($v = f\lambda$) of propagation of the propagating magnetic field.

The dynamical transition temperature T_c is measured here for a system of linear size $L = 100$. The systematic finite size analysis is not yet done. However, few results are

checked for smaller (say $L = 50$) system sizes. No appreciable change in T_c was observed.

4 Summary:

The dynamical response of two dimensional Ising ferromagnet in presence of a propagating magnetic field wave is studied by Monte Carlo simulation. A dynamical phase transition is observed. This dynamical phase transition is observed from the studies of the temperature variations of the dynamic order parameter, the derivative of the dynamic order parameter, the mean square deviation of the dynamic order parameter and the dynamic specific heat. All these studies indicate the dynamic phase transition and the transition temperatures are estimated.

For comparison the dynamic transition is also studied for a nonpropagating (sinusoidally oscillating in time but uniform over space) magnetic field. It is observed that the dynamic transition temperatures are different from that observed in the case of propagating magnetic field wave. It is observed, from figures 3, 4 and 5 that the propagating field wave causes the dynamical phase transition at lower temperature than that obtained from a non-propagating field of same amplitude. One may argue that since the propagating magnetic field makes, the strip magnetisation, a wave-like structure, the value of Q will be less than that for a non-propagating field of same amplitude and frequency at the same temperature. This would govern the transition to take place at lower temperature.

Here, the dependence of the transition temperature on the speed of propagation of the propagating magnetic field wave is studied briefly and it is observed that the transition takes place at higher temperature for the higher value of the speed of propagation. One may try to understand this fact in the following way: the increasing wavelength (or speed for a fixed frequency) simply makes the field more nearly homogeneous, approaching the infinite wavelength spatially homogeneous limit. It does appear that the transition for propagating field (with $h_0 = 0.5$) has shifted from $T = 1.50$ to that obtained for approximately spatially homogeneous case, i.e., $T = 1.68$.

The present observations, based on the Monte Carlo simulation, are reported here briefly. The dynamical phase boundary for propagating magnetic field wave is yet to be sketched and the dependence of the phase boundary on the frequency and wavelength of the propagating wave has to be determined. The finite size analysis and the detailed study of the behaviour of phase difference between propagating magnetic field wave and 'strip magnetisation' have to be done. It requires lot of computational efforts and will be reported later. The nonequilibrium dynamic phase transition in Ising ferromagnet, in the presence of propagating magnetic field wave, will become challenging in near future.

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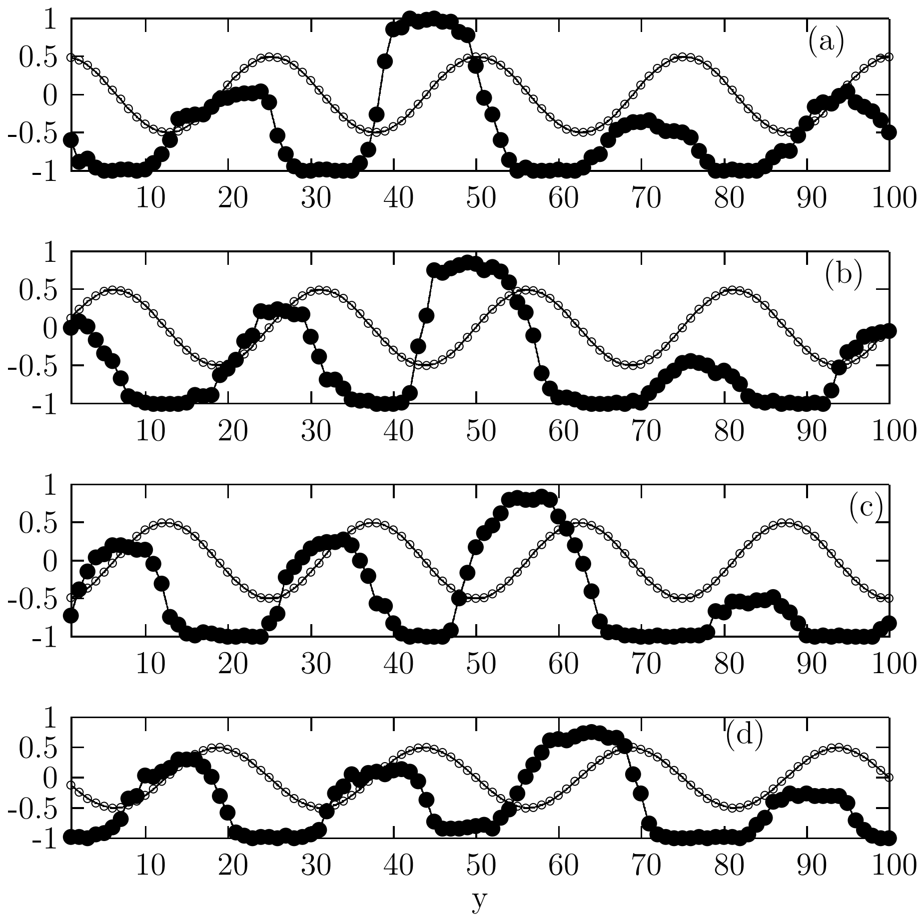

Fig-1. The spatio-temporal variations of propagating magnetic field wave ($h(y,t)$) and 'strip magnetisation' ($m(y,t)$) for $h_0 = 0.5$, $T = 1.50$ and $\lambda = 25$. The magnetic field and magnetisation are represented by open circles and bullets respectively. Continuous lines joining the data points act as guide to the eye. The plots for different times (t) are shown as follows: (a) $t = 100001$ MCSS, (b) $t = 100025$ MCSS, (c) $t = 100050$ MCSS and (d) $t = 100075$MCSS.

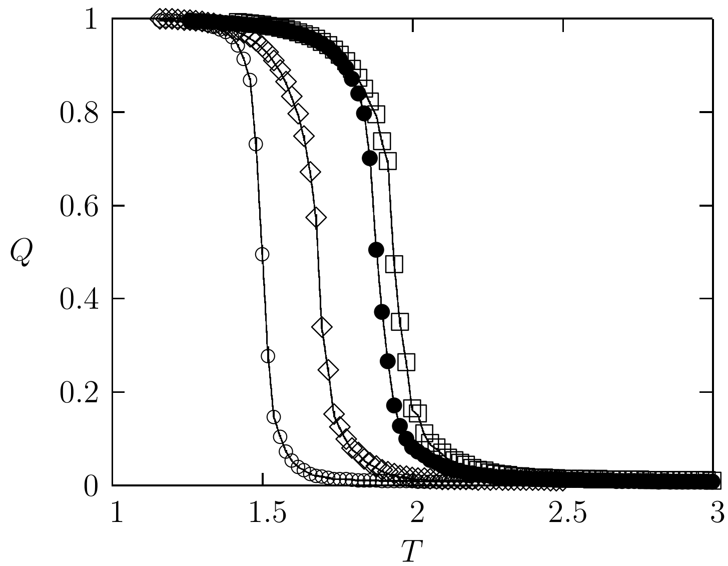

Fig.2. The temperature (T) variations of dynamic order parameter Q for different types of fields. (o) for propagating wave field with $h_0 = 0.5$, $\lambda = 25$ (and $\Delta_{max} = 0.216$), ($\bullet$) for propagating wave field with $h_0 = 0.3$, $\lambda = 25$ (and $\Delta_{max} = 0.197$) ($\diamond$) for non-propagating field with $h_0 = 0.5$ (and $\Delta_{max} = 0.167$), ($\square$) for non-propagating field with $h_0 = 0.3$ and ($\Delta_{max} = 0.200$). Here, the frequency $f = 0.01$ for both type of fields. Continuous lines just join the data points.

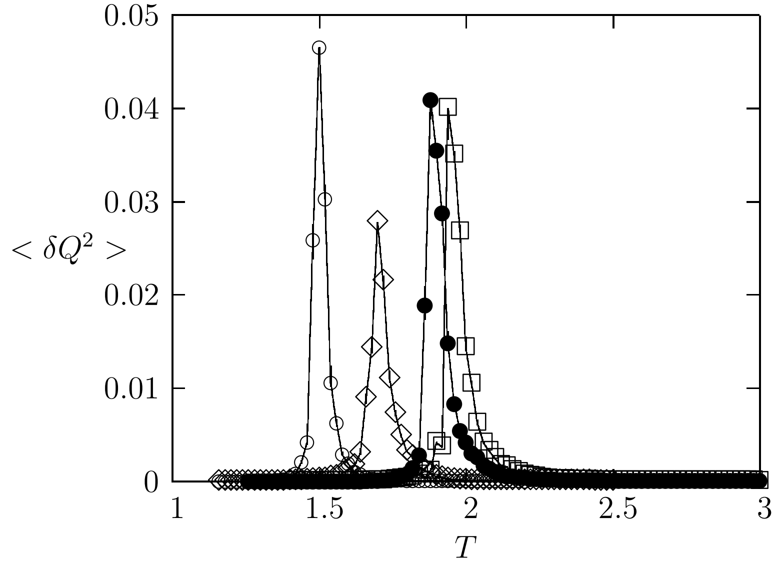

Fig.3. The temperature (T) variations of mean square deviation ($< \delta Q^2 >$) of the dynamic order parameter (Q) for different types of fields. (o) for propagating wave field with $h_0 = 0.5$ and $\lambda = 25$, ($\bullet$) for propagating wave field with $h_0 = 0.3$ and $\lambda = 25$, ($\diamond$) for non-propagating field with $h_0 = 0.5$, ($\Box$) for non-propagating field with $h_0 = 0.3$. Here, the frequency $f = 0.01$ for both type of fields. Continuous lines just join the data points.

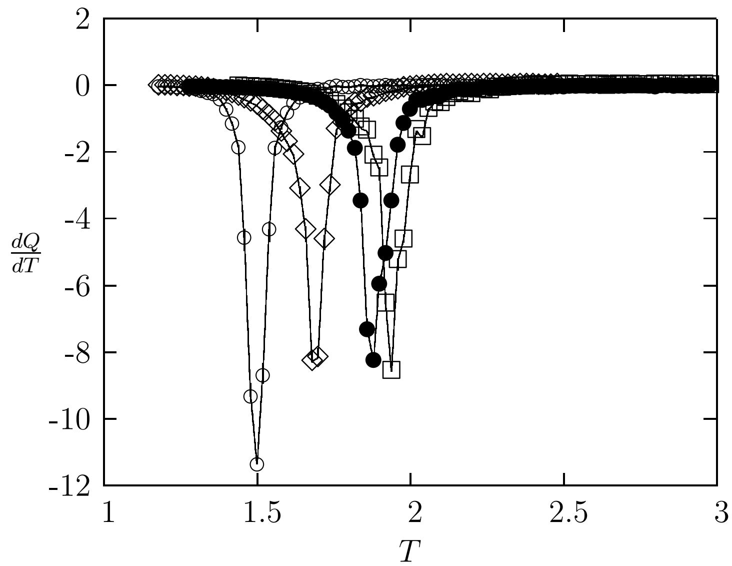

Fig.4. The temperature (T) variations of the derivative ($\frac{dQ}{dT}$) of the dynamic order parameter (Q) for different types of fields. (o) for propagating wave field with $h_0 = 0.5$ and $\lambda = 25$, ($\bullet$) for propagating wave field with $h_0 = 0.3$ and $\lambda = 25$, ($\diamond$) for non-propagating field with $h_0 = 0.5$, ($\square$) for non-propagating field with $h_0 = 0.3$. Here, the frequency $f = 0.01$ for both type of fields. Continuous lines just join the data points. Here, the error involved in calculating each data point is of the order of 0.0004.

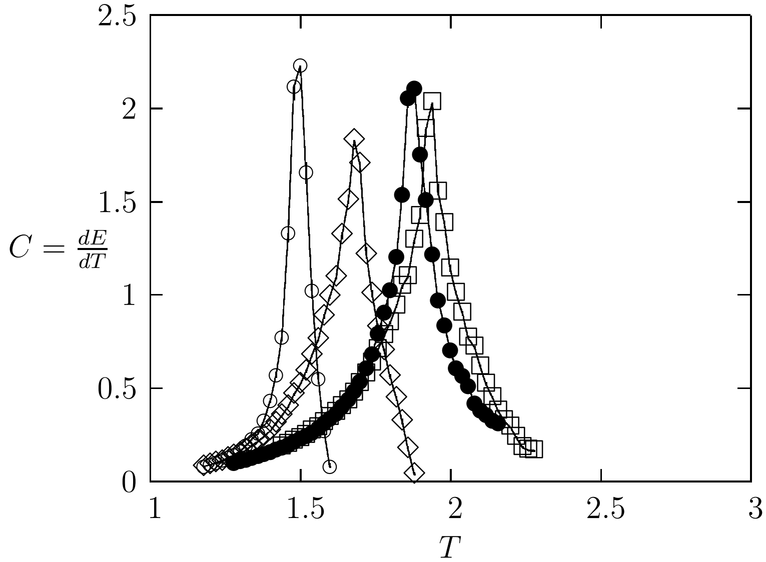

Fig.5. The temperature (T) variations of the derivative ($\frac{dE}{dT}$) of the dynamic energy i.e., dynamic specific heat, for different types of fields. (o) for propagating wave field with $h_0 = 0.5$ and $\lambda = 25$, ($\bullet$) for propagating wave field with $h_0 = 0.3$ and $\lambda = 25$, ($\diamond$) for non-propagating field with $h_0 = 0.5$, ($\Box$) for non-propagating field with $h_0 = 0.3$. Here, the frequency $f = 0.01$ for both type of fields. Continuous lines just join the data points. Here, the error involved in calculating each data point is of the order of 0.0004.

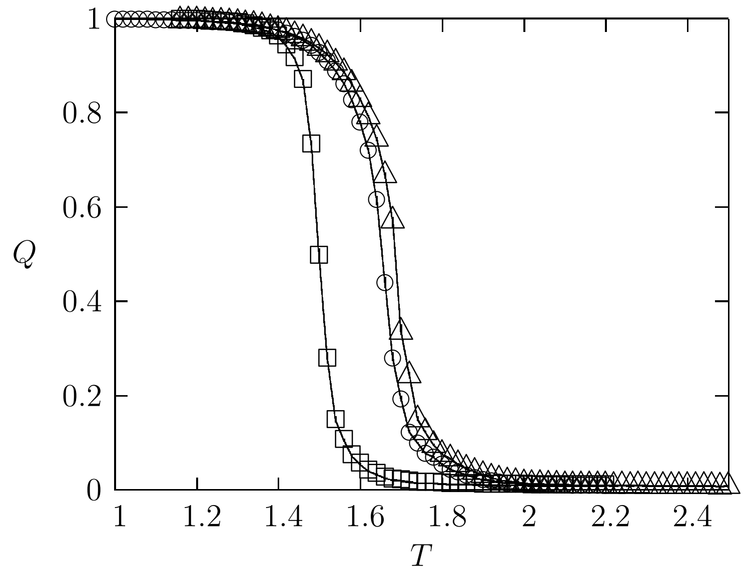

Fig.6. The temperature (T) variation of dynamic order parameter (Q) for propagating fields for two different velocities ($v = f\lambda$). ($\square$) represents $f = 0.01$, $\lambda = 25$, $h_0 = 0.5$ (and $\Delta_{max} = 0.216$) and (o) represents $f = 0.01$, $\lambda = 50$, $h_0 = 0.5$ (and $\Delta_{max} = 0.188$). For comparison, Q versus T is also plotted (represented by ($\triangle$)) for a non-propagating oscillating magnetic field with $f = 0.01$, $h_0 = 0.5$ (and $\Delta_{max} = 0.167$).